Statistical Simulation for Bifurcations

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Numerical simulation in critical domain (near a bifurcation) is considered to be unreliable since the results are reproduced unsatisfactorily. Because of large susceptibility there [1], smooth variations of parameters lead to disordered jumps in solution to the set of equations in question. Indeed, we deal with finite-difference equations, and the rounding, unavoidable in the course of calculations, acts as an uncontrolled noise. When close enough to the bifurcation, the variance of random pulsating is comparable to the mean value of the considered quantity as is the case in fully developed turbulence. To overcome this difficulty we propose addition of a small random function of time, e.g. white noise, to a constant source (pumping). Thereby the simulation approaches experimental situation. The solutions to Langevin equations [2] thus obtained are also random functions of time and require statistical reprocessing. Their properties, with the exception of intensity, are independent of the initial noise, and soft modes prevail in their spectrum. In the space of states, a complicated critical attractor appears instead of the limit point. We study this critical chaos [3,4], compare it to the dynamical one [5,6] and give examples of statistical description of bifurcations.

To forecast a future (in particular, predict catastrophes) the method of *compressive sensing* [7] was proposed [8,9]. There completely unknown set of equations of the general form

$$dx_i / dt = f_i(x_1, x_2, \dots), \quad i = 1,2,\dots$$

(i.e. with unknown right-hand sides) is reconstructed by using time series, in particular, experimental data. Right-hand sides of the equations are represented as power series in all the dependent variables. The coefficients of such expansion should be determined by compressive sensing. There is a code [7] for the determination. When the equations have been reconstructed one can find bifurcations (catastrophes) of their solutions. However, the authors [9] note difficulties appearing if the number of equations, i.e. variables, becomes large or time series are distorted by noise. In contrast, we suggest just adding the noise as a way to help predict catastrophes. We show that bifurcations of steady-state operation for a man-made or natural system can be predicted due to the onset of soft modes in the system's noise spectrum, see [10].

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